MATCHING OF DISCHARGE CIRCUIT WITH SHELL MOTION IN NONCYLINDRICAL Z-PINCH

V. V. Vikhrev

UDC 533.9.072 : 533.952.16

Model computations of the two-dimensional motion of a shell in a noncylindrical Z-pinch are carried out according to the scheme proposed in [1, 2]. In this two-dimensional snowplow model the configuration of the shell is determined at each instant of time and the shell is assumed to be infinitely thin. The computations are in good agreement with the available experimental data and permit one to predict the dimensions of the chamber and inductance of the discharge circuit necessary for an efficient use of the energy of the condenser battery.

1. Description of the Model. In this model it is assumed that the current shell in a noncylindrical Z-pinch is symmetric with respect to the z axis. This makes it possible to consider its motion only in the plane passing through the z axis (Fig. 1). Let λ be the curvilinear coordinate along the shell in this plane. One end of the shell corresponds to $\lambda = 0$, and the other to $\lambda = \lambda_f$, where $\lambda = \lambda_f(t)$ is the length of the entire shell. Let $\mu(t)$ denote the mass density along the shell. The total mass of the shell is equal to

$$M(t) = \int_0^{\lambda_f} \mu \, d\lambda$$

According to the "snowplow" model the gas is completely enclosed by the current shell and thereby increases its mass. If the shell is assumed to be sufficiently thin, then

$$d\mu / dt = 2\pi R \rho (\mathbf{n} \cdot \mathbf{V}) \tag{1.1}$$

where ρ is the initial density of the gas, $R(\lambda, t)$ is the radius of the shell, $\mathbf{V}(\lambda, t)$ is the velocity of the shell, and $\mathbf{n}(\lambda, t)$ is the normal to the current shell directed into the plasma.

The equation describing the motion of an element of mass has the form

$$\frac{d\mu \mathbf{V}}{dt} = \frac{I^2}{c^2 R} \mathbf{n} \tag{1.2}$$

The magnitude of the current $I(t)$ is determined by the equation of the circuit

$$U_0 - \frac{1}{c} \int_0^t I \, d\lambda = \frac{dLI}{dt} \tag{1.3}$$

Here U_0 is the initial voltage, c is the capacitance of the condenser battery, and $L = L_0 + L_1$, where L_0 is the initial inductance and L_1 is the variable inductance of the shell.

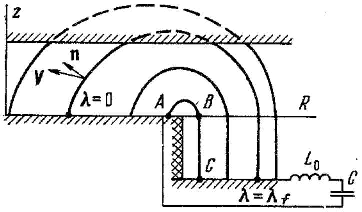

Fig. 1

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 160-162, March-April, 1973. Original article submitted June 27, 1972.

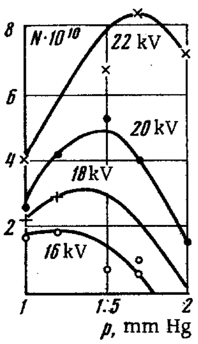

Fig. 2

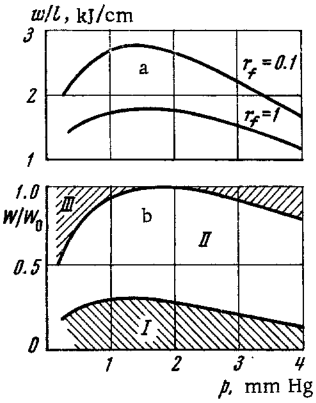

Fig. 3

2. Computational Scheme. The equations obtained above were solved numerically on a computer. The initial configuration of the shell was specified in the form of a semicircle AB and the straight line segment BC (Fig. 1). Arc AB was divided uniformly into not more than 200 computational segments. An increase of the number of computational points or their nonuniform distribution on arc AB causes instability of the computational scheme.

The impact of the shell on the upper lid of the discharge chamber was taken into consideration by the fact that in the equation of the circuit the inductance of the shell corresponding to the space above the lid of the chamber was not considered.

3. Criterion of Matching. In the study of Z-pinches a simple criterion of good operation of the equipment is the number of neutrons forming after the discharge as a result of thermonuclear reaction. It depends simultaneously on such quantities as the density, temperature, volume, and life time of the plasma. The dependence of the neutron yield on the voltage at the battery and on deuterium pressure in the chamber, obtained on "noncylindrical Z-pinch" equipment [3], is shown in Fig. 2. It is characteristic that with the increase of the voltage the regime with largest neutron yield gets displaced toward larger pressures. This dependence is a consequence of the conditions of matching of the discharge time of the battery with the time of motion of the current shell, and it points out the significance of these conditions.

In the model used here, it is difficult to determine the temperature and the final radius of the pinch, since the structure of the shell is not considered. A sufficiently good criterion of matching of the external circuit with the motion of the current shell may be the kinetic energy per unit height of the pinch in the zone of focus during its compression to a certain sufficiently small "final" radius r_f (we shall denote this energy as w/l). Later, at the time of maximum compression, this energy gets converted into thermal energy, and therefore the quantity w/l is characterized by NT, where N is the number of particles per unit height of the pinch and T is the plasma temperature.

Since within the framework of the present model it is not possible to determine the final radius r_f, the results of the computations are given for two values of r_f differing by a factor of 10. Numerical computations showed that the determination of the optimum has a weak dependence on r_f so that the exact value of r_f is not important.

4. Effect of Gas Pressure on Matching. In Fig. 3a the computed energy per unit height of the pinch w/l is shown as a function of the initial pressure p of deuterium in the chamber for the typical regime of the "noncylindrical Z-pinch" equipment [3] ($U_0 = 18$ kV, $W = 93$ kJ, $L_0 = 30$ nH).

It is evident that for the investigated variant of the chamber (anode radius 33 cm) the equipment operates in the matched regime (i.e., w/l is maximum) for $p = 1.5$ mm Hg D_2. This result does not depend on the choice of r_f. In the "noncylindrical Z-pinch" equipment the maximum neutron yield under the given conditions was observed at a pressure of 1.3 mm Hg D_2 (Fig. 2).

The relationship among the individual forms of energy at the time of cumulation at the axis (I – magnetic energy, II – energy in the plasma, II – energy in the condenser battery) is shown in Fig. 3b. In the matched regime the energy remaining in the condenser battery does not exceed 1% of the initial stored energy. This means that on varying p, the maximum of w/l is attained for the most complete utilization of the condenser battery.

5. Effect of Inductance of External Circuit on Matching. The energy per unit height of the pinch w/l is shown in Fig. 4 as a function of the inductance of the external circuit L_0 for the typical regime of the equipment ($C = 576$ μF, $U_0 = 18$ kV, $R = 33$ cm, $p = 1$ mm Hg) (I for $r_f = 0.1$ cm, II for $r_f = 1$ cm). The presence of the maximum is explained by the fact that for a very large inductance a significant part of the energy remains in the external circuit, while for too small values of L_0 much of the energy is given out at the very beginning of the discharge, and the shell gets "weak" by the time it reaches the axis of the chamber. The value of L_0 at which w/l is maximum is related to the inductance of the shell in the compressed state L_f through the relation

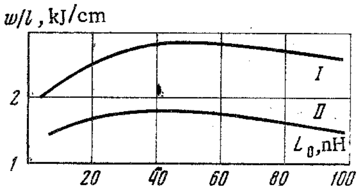

Fig. 4

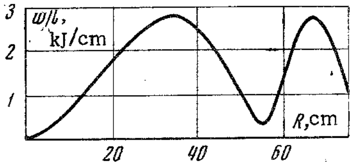

Fig. 5

$$L_0 \approx 1/2\, L_f$$

Thus, for example, in Fig. 4 (curve I) the maximum of w/l occurs for L_0=nH, while the inductance of the shell at the time of compression with $r_f = 0.1$ cm is 108 nH. For the compression of the shell to r_f = 1 cm its inductance is equal to 90 nH, and the matched regime is observed for L_0=40 nH (curve II in Fig. 4).

Due to the fact that in the region of the maximum the dependence of the energy per unit height of the pinch on L_0 and p is weak, in practice it is convenient to operate at inductances L_0 of the circuit somewhat smaller than the optimum, which lowers the voltage in the chamber at the time of compression, and at smaller p, which gives a somewhat larger rate of convergence of the shell than in the matched regime.

6. Fffect of Dimensions of Chamber on Matching. The main geometrical parameter affecting the matching of the system is the radius of the inner electrode R. The dependence of w/l on R is shown in Fig. 5 for C = 576 μF, U_0 = 18 kV, and p = 1 mm Hg. It is evident that for given parameters of the discharge circuit and for a given pressure p the equipment operates in the matched regime for the radius of the node equal to 30–40 cm. The second maximum on the curve of w/l corresponds to the case where the maximum compression occurs in the second half-period of the discharge current. In this case, the radius R of the node must be ~65 cm. However, usually the equipment is not operated at the second maximum due to the decrease of the rate of compression.

Computations show that for a chamber with the height of the insulator equal to 12 cm and the distance between the node and the lid of the chamber equal to 14 cm, the matched regime in deuterium is obtained for the radius of the chamber

$$R = (0.088\, CU_0 p^{-1/2})^{0.53} \tag{6.1}$$

where R is in cm, C in μF, U_0 in kV, and p in mm Hg. The change of the height of the electrode and the distance between the electrodes does not have a significant effect on the matching condition. It follows from (6.1) that in an equipment with the battery energy equal to 10 MJ and the voltage equal to 50 kV in order to obtain matching at 1 mm Hg D_2 the radius of the node of the chamber must be equal to ~2.5 m.

In conclusion the author thanks N. V. Filippov and V. V. Prut for discussion and help.

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